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New strategies to extract weak phases from neutral B decays

Robert Fleischer

Theory Division, CERN, CH-1211 Geneva 23, Switzerland

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Abstract. We discuss new, theoretically clean strategies to determine the angle γ of the unitarity triangle from $B_d \to DK_{\mathrm{S(L)}}$, $B_s \to D\eta^{(\prime)}, D\phi, ...$ decays, and point out that $B_s \to DK_{\mathrm{S(L)}}$ and $B_d \to D\pi^0, D\rho^0, ...$ modes allow very interesting determinations of the B^0_q–$\overline{B^0_q}$ mixing phases ϕ_s and ϕ_d, respectively. Their colour-allowed counterparts $B_s \to D_s^{(*)\pm}K^\mp, ...$ and $B_d \to D^{(*)\pm}\pi^\mp, ...$ also offer new methods to probe γ.

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1 Introduction

The time-dependent CP asymmetries of neutral B_q-meson decays ($q \in \{d,s\}$) into CP eigenstates, which satisfy $(\mathcal{CP})|f\rangle = \pm|f\rangle$, provide valuable information [1]:

$$\frac{\Gamma(B^0_q(t)\to f)-\Gamma(\overline{B^0_q}(t)\to f)}{\Gamma(B^0_q(t)\to f)+\Gamma(\overline{B^0_q}(t)\to f)} = \frac{\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}\cos(\Delta M_q t)+\mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}}\sin(\Delta M_q t)}{\cosh(\Delta\Gamma_q t/2)-\mathcal{A}_{\Delta\Gamma}\sinh(\Delta\Gamma_q t/2)}. \quad (1)$$

Here the CP-violating observables

$$\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}} \equiv \frac{1-\left|\xi_f^{(q)}\right|^2}{1+\left|\xi_f^{(q)}\right|^2} \quad \text{and} \quad \mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}} \equiv \frac{2\,\mathrm{Im}\,\xi_f^{(q)}}{1+\left|\xi_f^{(q)}\right|^2} \quad (2)$$

originate from "direct" and "mixing-induced" CP violation, respectively, and are governed by

$$\xi_f^{(q)} = -e^{-i\phi_q}\left[\frac{A(\overline{B^0_q}\to f)}{A(B^0_q\to f)}\right], \quad (3)$$

where

$$\phi_q \overset{\mathrm{SM}}{=} 2\arg(V_{tq}^*V_{tb}) = \begin{cases} +2\beta & (q=d) \\ -2\lambda^2\eta & (q=s) \end{cases} \quad (4)$$

is the CP-violating weak B^0_q–$\overline{B^0_q}$ mixing phase. The width difference $\Delta\Gamma_q$, which may be sizeable in the $q=s$ case, offers another observable $\mathcal{A}_{\Delta\Gamma}$, which is, however, not independent from those in (2), and can be extracted from the following "untagged" rates:

$$\langle\Gamma(B_q(t)\to f)\rangle \equiv \Gamma(B^0_q(t)\to f)+\Gamma(\overline{B^0_q}(t)\to f) \propto \left[\cosh(\Delta\Gamma_q t/2)-\mathcal{A}_{\Delta\Gamma}\sinh(\Delta\Gamma_q t/2)\right]e^{-\Gamma_q t}. \quad (5)$$

2 $B_d \to DK_{\mathrm{S(L)}}$, $B_s \to D\eta^{(\prime)}, D\phi, ...$ and $B_s \to DK_{\mathrm{S(L)}}$, $B_d \to D\pi^0, D\rho^0, ...$

Let us consider in this section $B^0_q \to D^0 f_r$ transitions, where $r \in \{s,d\}$ distinguishes between $b \to Ds$ and $b \to Dd$ processes [2,3]. If we require $(\mathcal{CP})|f_r\rangle = \eta_{\mathrm{CP}}^{f_r}|f_r\rangle$, B^0_q and $\overline{B^0_q}$ mesons may both decay into $D^0 f_r$, thereby leading to interference effects between B^0_q–$\overline{B^0_q}$ mixing and decay processes, which involve the weak phase $\phi_q+\gamma$:

- For $r=s$, i.e. $B_d \to DK_{\mathrm{S(L)}}$, $B_s \to D\eta^{(\prime)}, D\phi$, ..., these effects are governed by a hadronic parameter $x_{f_s}e^{i\delta_{f_s}} \propto R_b \approx 0.4$, and are hence favourably large.
- For $r=d$, i.e. $B_s \to DK_{\mathrm{S(L)}}$, $B_d \to D\pi^0, D\rho^0$ ..., these effects are tiny because of $x_{f_d}e^{i\delta_{f_d}} \propto -\lambda^2 R_b \approx -0.02$.

2.1 $B_d \to DK_{\mathrm{S(L)}}$, $B_s \to D\eta^{(\prime)}, D\phi, ...$

Let us first focus on $r=s$. If we make use of the CP eigenstates $D_\pm$ of the neutral D-meson system satisfying $(\mathcal{CP})|D_\pm\rangle = \pm|D_\pm\rangle$, we obtain additional interference effects between $B^0_q \to D^0 f_s$ and $B^0_q \to \overline{D^0} f_s$ at the decay-amplitude level, which involve γ. The most straightforward observable we may measure is the "untagged" rate

$$\begin{aligned}&\langle\Gamma(B_q(t)\to D_\pm f_s)\rangle \equiv \\ &\Gamma(B^0_q(t)\to D_\pm f_s)+\Gamma(\overline{B^0_q}(t)\to D_\pm f_s) \\ &\overset{\Delta\Gamma_q=0}{=} \left[\Gamma(B^0_q\to D_\pm f_s)+\Gamma(\overline{B^0_q}\to D_\pm f_s)\right]e^{-\Gamma_q t} \\ &\equiv \langle\Gamma(B_q\to D_\pm f_s)\rangle e^{-\Gamma_q t},\end{aligned} \quad (6)$$

providing the following "untagged" rate asymmetry:

$$\Gamma_{+-}^{f_s} \equiv \frac{\langle\Gamma(B_q\to D_+ f_s)\rangle-\langle\Gamma(B_q\to D_- f_s)\rangle}{\langle\Gamma(B_q\to D_+ f_s)\rangle+\langle\Gamma(B_q\to D_- f_s)\rangle}. \quad (7)$$

Interestingly, already this quantity offers valuable information on γ, since bounds on this angle are implied by

$$|\cos\gamma| \geq |\Gamma^{f_s}_{+-}|. \quad (8)$$

Moreover, if we fix the sign of $\cos\delta_{f_s}$ with the help of the factorization approach, we obtain

$$\mathrm{sgn}(\cos\gamma) = -\mathrm{sgn}(\Gamma^{f_s}_{+-}), \quad (9)$$

i.e. we may decide whether γ is smaller or larger than 90°. If we employ, in addition, the mixing-induced observables $S^{f_s}_{\pm} \equiv \mathcal{A}^{\mathrm{mix}}_{\mathrm{CP}}(B_q \to D_{\pm} f_s)$, we may determine γ. To this end, it is convenient to introduce the quantities

$$\langle S_{f_s}\rangle_{\pm} \equiv \frac{S^{f_s}_{+} \pm S^{f_s}_{-}}{2}. \quad (10)$$

Expressing the $\langle S_{f_s}\rangle_{\pm}$ in terms of the $B_q \to D_{\pm} f_s$ decay parameters gives rather complicated formulae. However, complementing the $\langle S_{f_s}\rangle_{\pm}$ with $\Gamma^{f_s}_{+-}$ yields

$$\tan\gamma\cos\phi_q = \left[\frac{\eta_{f_s}\langle S_{f_s}\rangle_{+}}{\Gamma^{f_s}_{+-}}\right] + \left[\eta_{f_s}\langle S_{f_s}\rangle_{-} - \sin\phi_q\right], \quad (11)$$

where $\eta_{f_s} \equiv (-1)^L \eta^{f_s}_{\mathrm{CP}}$, with L denoting the Df_s angular momentum [2]. If we use this simple – but *exact* – relation, we obtain the twofold solution $\gamma = \gamma_1 \vee \gamma_2$, with $\gamma_1 \in [0°, 180°]$ and $\gamma_2 = \gamma_1 + 180°$. Since $\cos\gamma_1$ and $\cos\gamma_2$ have opposite signs, (9) allows us to fix γ *unambiguously*. Another advantage of (11) is that $\langle S_{f_s}\rangle_{+}$ and $\Gamma^{f_s}_{+-}$ are both proportional to $x_{f_s} \approx 0.4$, so that the first term in square brackets is of $\mathcal{O}(1)$, whereas the second one is of $\mathcal{O}(x^2_{f_s})$, hence playing a minor rôle. In order to extract γ, we may also employ D decays into CP non-eigenstates f_{NE}, where we have to deal with complications originating from $D^0, \overline{D^0} \to f_{\mathrm{NE}}$ interference effects [4]. Also in this case, $\Gamma^{f_s}_{+-}$ is a very powerful ingredient, offering an efficient, *analytical* strategy to include these interference effects in the extraction of γ [3].

2.2 $B_s \to DK_{\mathrm{S(L)}}$, $B_d \to D\pi^0, D\rho^0, \ldots$

The $r = d$ case also has interesting features. It corresponds to $B_s \to DK_{\mathrm{S(L)}}$, $B_d \to D\pi^0, D\rho^0$ ... decays, which can be described through the same formulae as their $r = s$ counterparts. Since the relevant interference effects are governed by $x_{f_d} \approx -0.02$, these channels are not as attractive for the extraction of γ as the $r = s$ modes. On the other hand, the relation

$$\eta_{f_d}\langle S_{f_d}\rangle_{-} = \sin\phi_q + \mathcal{O}(x^2_{f_d}) = \sin\phi_q + \mathcal{O}(4\times 10^{-4}) \quad (12)$$

offers very interesting determinations of $\sin\phi_q$ [2]. Following this avenue, there are no penguin uncertainties, and the theoretical accuracy is one order of magnitude better than in the "conventional" $B_d \to J/\psi K_{\mathrm{S}}$, $B_s \to J/\psi\phi$ strategies. In particular, $\phi_s^{\mathrm{SM}} = -2\lambda^2\eta$ could, in principle, be determined with a theoretical uncertainty of only $\mathcal{O}(1\%)$, in contrast to the extraction from the $B_s \to J/\psi\phi$ angular distribution, which suffers from generic penguin uncertainties at the 10% level.

3 $B_s \to D_s^{(*)\pm}K^{\mp}, \ldots$ and $B_d \to D^{(*)\pm}\pi^{\mp}, \ldots$

Let us now consider the colour-allowed counterparts of the $B_q \to Df_q$ modes discussed above, which we may write generically as $B_q \to D_q\overline{u}_q$ [5]. The characteristic feature of these transitions is that both a B^0_q and a $\overline{B^0_q}$ meson may decay into $D_q\overline{u}_q$, thereby leading to interference between B^0_q–$\overline{B^0_q}$ mixing and decay processes, which involve the weak phase $\phi_q + \gamma$:

- In the case of $q = s$, i.e. $D_s \in \{D_s^+, D_s^{*+}, \ldots\}$ and $u_s \in \{K^+, K^{*+}, \ldots\}$, these effects are favourably large as they are governed by $x_s e^{i\delta_s} \propto R_b \approx 0.4$.
- In the case of $q = d$, i.e. $D_d \in \{D^+, D^{*+}, \ldots\}$ and $u_d \in \{\pi^+, \rho^+, \ldots\}$, the interference effects are described by $x_d e^{i\delta_d} \propto -\lambda^2 R_b \approx -0.02$, and hence are tiny.

We shall only consider $B_q \to D_q\overline{u}_q$ modes, where at least one of the $D_q, \overline{u}_q$ states is a pseudoscalar meson; otherwise a complicated angular analysis has to be performed.

It is well known that such decays allow determinations of the weak phases $\phi_q + \gamma$, where the "conventional" approach works as follows [6,7]: if we measure the observables $C(B_q \to D_q\overline{u}_q) \equiv C_q$ and $C(B_q \to \overline{D}_q u_q) \equiv \overline{C}_q$ provided by the $\cos(\Delta M_q t)$ pieces of the time-dependent rate asymmetries, we may determine x_q from terms entering at the x_q^2 level. In the case of $q = s$, we have $x_s = \mathcal{O}(R_b)$, implying $x_s^2 = \mathcal{O}(0.16)$, so that this may actually be possible, although challenging. On the other hand, $x_d = \mathcal{O}(-\lambda^2 R_b)$ is doubly Cabibbo-suppressed. Although it should be possible to resolve terms of $\mathcal{O}(x_d)$, this will be impossible for the vanishingly small $x_d^2 = \mathcal{O}(0.0004)$ terms, so that other approaches to fix x_d are required [6]. In order to extract $\phi_q + \gamma$, the mixing-induced observables $S(B_q \to D_q\overline{u}_q) \equiv S_q$ and $S(B_q \to \overline{D}_q u_q) \equiv \overline{S}_q$ associated with the $\sin(\Delta M_q t)$ terms of the time-dependent rate asymmetries must be measured, where it is convenient to introduce

$$\langle S_q\rangle_{\pm} \equiv \frac{\overline{S}_q \pm S_q}{2}. \quad (13)$$

If we assume that x_q is known, we may consider

$$s_{+} \equiv (-1)^L \left[\frac{1 + x_q^2}{2x_q}\right]\langle S_q\rangle_{+} = +\cos\delta_q \sin(\phi_q + \gamma) \quad (14)$$

$$s_{-} \equiv (-1)^L \left[\frac{1 + x_q^2}{2x_q}\right]\langle S_q\rangle_{-} = -\sin\delta_q \cos(\phi_q + \gamma), \quad (15)$$

yielding

$$\sin^2(\phi_q + \gamma) = \frac{1 + s_+^2 - s_-^2}{2} \pm \sqrt{\frac{(1 + s_+^2 - s_-^2)^2 - 4s_+^2}{4}}, \quad (16)$$

which implies an eightfold solution for $\phi_q + \gamma$. If we fix the sign of $\cos\delta_q$ with the help of factorization, a fourfold discrete ambiguity emerges. Note that this assumption allows us to extract also the sign of $\sin(\phi_q + \gamma)$ from $\langle S_q\rangle_{+}$, which is of particular interest, as discussed in [5]. To this end, the factor $(-1)^L$, where L is the $D_q\overline{u}_q$ angular momentum, has to be properly taken into account.

Let us now discuss the new strategies to explore the $B_q \to D_q \overline{u}_q$ modes proposed in [5]. If $\Delta\Gamma_s$ is sizeable, the time-dependent "untagged" rates introduced in (5)

$$\langle \Gamma(B_q(t) \to D_q\overline{u}_q)\rangle = \langle \Gamma(B_q \to D_q\overline{u}_q)\rangle e^{-\Gamma_q t} \times \left[\cosh(\Delta\Gamma_q t/2) - \mathcal{A}_{\Delta\Gamma}(B_q \to D_q\overline{u}_q)\sinh(\Delta\Gamma_q t/2)\right] \quad (17)$$

and their CP conjugates provide $\mathcal{A}_{\Delta\Gamma}(B_s \to D_s\overline{u}_s) \equiv \mathcal{A}_{\Delta\Gamma_s}$ and $\mathcal{A}_{\Delta\Gamma}(B_s \to \overline{D}_s u_s) \equiv \overline{\mathcal{A}}_{\Delta\Gamma_s}$, which yield

$$\tan(\phi_s+\gamma) = -\left[\frac{\langle S_s\rangle_+}{\langle \mathcal{A}_{\Delta\Gamma_s}\rangle_+}\right] = +\left[\frac{\langle \mathcal{A}_{\Delta\Gamma_s}\rangle_-}{\langle S_s\rangle_-}\right], \quad (18)$$

where the $\langle \mathcal{A}_{\Delta\Gamma_s}\rangle_\pm$ are defined in analogy to (13). These relations allow an *unambiguous* extraction of $\phi_s+\gamma$ if we fix again the sign of $\cos\delta_q$ through factorization. Another important advantage of (18) is that we do *not* have to rely on $\mathcal{O}(x_s^2)$ terms, as $\langle S_s\rangle_\pm$ and $\langle \mathcal{A}_{\Delta\Gamma_s}\rangle_\pm$ are proportional to x_s. On the other hand, we need a sizeable value of $\Delta\Gamma_s$. Measurements of untagged rates are also very useful in the case of vanishingly small $\Delta\Gamma_q$, since the "unevolved" untagged rates in (17) offer various interesting strategies to determine x_q from the ratio of $\langle\Gamma(B_q \to D_q\overline{u}_q)\rangle + \langle\Gamma(B_q \to \overline{D}_q u_q)\rangle$ to CP-averaged rates of appropriate $B^\pm$ or flavour-specific B_q decays.

If we keep the hadronic parameter x_q and the associated strong phase δ_q as "unknown", free parameters in the expressions for the $\langle S_q\rangle_\pm$, we obtain

$$|\sin(\phi_q+\gamma)| \geq |\langle S_q\rangle_+|, \quad |\cos(\phi_q+\gamma)| \geq |\langle S_q\rangle_-|, \quad (19)$$

which can straightforwardly be converted into bounds on $\phi_q+\gamma$. If x_q is known, stronger constraints are implied by

$$|\sin(\phi_q+\gamma)| \geq |s_+|, \quad |\cos(\phi_q+\gamma)| \geq |s_-|. \quad (20)$$

Once s_+ and s_- are known, we may of course determine $\phi_q+\gamma$ through the "conventional" approach, using (16). However, the bounds following from (20) provide essentially the same information and are much simpler to implement. Moreover, as discussed in detail in [5] for several examples, the bounds following from the B_s and B_d modes may be highly complementary, thereby providing particularly narrow, theoretically clean ranges for γ.

Let us now further exploit the complementarity between the $B_s^0 \to D_s^{(*)+}K^-$ and $B_d^0 \to D^{(*)+}\pi^-$ modes. If we look at their decay topologies, we observe that these channels are related to each other through an interchange of all down and strange quarks. Consequently, the U-spin flavour symmetry of strong interactions implies $a_s = a_d$ and $\delta_s = \delta_d$, where $a_s = x_s/R_b$ and $a_d = -x_d/(\lambda^2 R_b)$ are the ratios of hadronic matrix elements entering x_s and x_d, respectively. There are various possibilities to implement these relations. A particularly simple picture emerges if we assume that $a_s = a_d$ *and* $\delta_s = \delta_d$, which yields

$$\tan\gamma = -\left[\frac{\sin\phi_d - S\sin\phi_s}{\cos\phi_d - S\cos\phi_s}\right] \stackrel{\phi_s=0^\circ}{=} -\left[\frac{\sin\phi_d}{\cos\phi_d - S}\right]. \quad (21)$$

Here we have introduced

$$S = -R\left[\frac{\langle S_d\rangle_+}{\langle S_s\rangle_+}\right] \text{ with } R = \left(\frac{1-\lambda^2}{\lambda^2}\right)\left[\frac{1}{1+x_s^2}\right], \quad (22)$$

where R can be fixed from untagged B_s rates through

$$R = \left(\frac{f_K}{f_\pi}\right)^2 \frac{\Gamma(\overline{B^0_s} \to D_s^{(*)+}\pi^-) + \Gamma(B^0_s \to D_s^{(*)-}\pi^+)}{\langle\Gamma(B_s \to D_s^{(*)+}K^-)\rangle + \langle\Gamma(B_s \to D_s^{(*)-}K^+)\rangle}. \quad (23)$$

Alternatively, we may *only* assume that $\delta_s = \delta_d$ *or* that $a_s = a_d$. Apart from features related to multiple discrete ambiguities, the most important advantage with respect to the "conventional" approach is that the experimental resolution of the x_q^2 terms is not required. In particular, x_d does *not* have to be fixed, and x_s may only enter through a $1+x_s^2$ correction, which can straightforwardly be determined through untagged B_s rate measurements. In the most refined implementation of this strategy, the measurement of x_d/x_s would *only* be interesting for the inclusion of U-spin-breaking effects in a_d/a_s. Moreover, we may obtain interesting insights into hadron dynamics and U-spin-breaking effects.

4 Conclusions

We have discussed new strategies to explore CP violation through neutral B_q decays. In the first part, we have shown that $B_d \to DK_{\mathrm{S(L)}}$, $B_s \to D\eta^{(\prime)}, D\phi$, ... modes provide theoretically clean, efficient and unambiguous extractions of $\tan\gamma$ if we combine an "untagged" rate asymmetry with mixing-induced observables. On the other hand, their $B_s \to D_\pm K_{\mathrm{S(L)}}$, $B_d \to D_\pm\pi^0, D_\pm\rho^0$, ... counterparts are not as attractive for the determination of γ, but allow extremely clean extractions of the mixing phases ϕ_s and ϕ_d, which may be particularly interesting for the ϕ_s case. In the second part, we have discussed interesting new aspects of $B_s \to D_s^{(*)\pm}K^\mp$, ... and $B_d \to D^{(*)\pm}\pi^\mp$, ... decays. The observables of these modes provide clean bounds on $\phi_q+\gamma$, where the resulting ranges for γ may be highly complementary in the B_s and B_d cases, thereby yielding stringent constraints on γ. Moreover, it is of great advantage to combine the $B_d \to D^{(*)\pm}\pi^\mp$ modes with their U-spin counterparts $B_s \to D_s^{(*)\pm}K^\mp$, allowing us to overcome the main problems of the "conventional" strategies to deal with these modes. We strongly encourage detailed feasibility studies of these new strategies.

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